

## Applied Bayesian Statistics

Herbert Hoijtink, Irene Klugkist, Ellen Hamaker, Charlotte Rietbergen, Joran Jongerling  
and Hennie Boeije

Department of Methods and Statistics  
P.O.Box 80140  
3508 TC Utrecht

[h.hojtink@uu.nl](mailto:h.hojtink@uu.nl); [i.klugkist@uu.nl](mailto:i.klugkist@uu.nl); [e.l.hamaker@uu.nl](mailto:e.l.hamaker@uu.nl); [c.rietbergen@uu.nl](mailto:c.rietbergen@uu.nl),  
[j.jongerling@uu.nl](mailto:j.jongerling@uu.nl), [h.boeije@uu.nl](mailto:h.boeije@uu.nl)

### Abstract

The interest in Bayesian statistic has been increasing for the last two decades both among statisticians and researchers who cannot properly analyze their data using methods based on classical statistics. This course will provide a sound basis in Bayesian statistics for those who

- Want to understand what Bayesian statistics is about
- Want to use Bayesian statistics to build and evaluate statistical models
- Want to get hands on experience with Bayesian statistics in WinBugs/Openbugs

### Target Group

The course is aimed at researchers who not only work with statistical tools, but are also interested in the development and evaluation of statistical tools. Among these are psychometricians, sociometricians, epidemiologists and statisticians. The only requirement is familiarity with the following concepts: the likelihood function, the p-value, analysis of variance, and multiple regression.

### Course Data, Course Fee and Course Materials

The course will be given 7 through 11 May 2012. Each day will start at 9.00 and finish at 17.00. The location will be in Utrecht and will be announced in the first week of May 2012.

- IOPS PhD's can enroll in this course until March 1<sup>st</sup> 2012. The course is free for this group, 12 places have been reserved for this group. Participation will be based on the order enrollment.
- Master Students from UU epidemiology can enroll in this course until April 1<sup>st</sup> 2012. The course is free for this group, 12 places have been reserved for this group. Participation will be based on the order of enrollment.
- If the group has not exceeded 24 participants, others can enroll in this course after April 1<sup>st</sup> 2012. The course fee for others is Euro 750.--.

Participants can enroll by sending an e-mail to [h.hojtink@uu.nl](mailto:h.hojtink@uu.nl).

A reader containing the most important literature and course materials will be made available at <http://tinyurl.com/hoijtink> in the first week of May 2012. The reader will contain the papers and chapters by Hoijtink and Klugkist that are listed in the reference list below. The other papers are freely available via the library of most universities and can be down-loaded for further reading. The four books listed in the reference list can also be considered further reading, and can be bought in most academic book shops. Participants are well-advised to buy the WinBUGS book by Ntzoufras.

Participants are required to bring their own lap-top on which ConfirmatoryAnova (go to <http://tinyurl.com/hoijtink>, click on the link at the top of the page and the page containing this software package will appear) and OpenBugs (<http://www.openbugs.info/w/>) have to be installed.

## Grading

Students who want to obtain a grade and credit points (Epidemiology Master Students 1.5 EC, IOPS PhD students 2 EC) for this course have to attend the grading session on Friday. Each student can choose one of the following options:

- Present a theorem and defend it. This could, for example, be a position in the controversy between classical and Bayesian statistics
- Present a poster (which may be hand written – paper and markers will be available at all lab meetings) about an exercise executed during this course. Note that the poster may be sketchy, and used as support material for the story you want to tell.
- Besides theorem and poster other ideas are appreciated. However, verify with the instructor before you start working on it.

Whichever option you choose, you have 5 minutes to deal with it. Note that preparation for the Grading Session has to take place during the lab-meetings.

### **Day 1: Applied Bayesian Statistics: the Evaluation of Informative Hypotheses.**

Using empirical data sets and associated research questions, the potential of applied Bayesian statistics will be illustrated. This day consists of four lectures and a lab meeting.

Lecture 1: Empirical data sets and associated research questions will be introduced. It will be shown how the authors of the papers from which data and research questions originated tried to use classical statistics (predominantly hypothesis testing) to answer their research questions.

Lecture 2: The Bayesian alternative for hypothesis testing (the Bayes factor) will be introduced in a non-technical manner. Attention will be given to the following concepts: prior distribution, likelihood, posterior distribution, marginal likelihood, the Bayes factor and posterior probabilities. An elaboration of these concepts will be provided on Day 2 and Day 4.

Lecture 3: Using the data and research questions introduced in Lecture 1 it will be elaborated how the Bayes factor can be used to answer the research questions. Furthermore, a comparison will be made between classical hypothesis testing using p-values and model selection using the Bayes factor. Advantages and disadvantages of both approaches will be discussed.

Lecture 4: Translating prior knowledge into hypotheses.

Lab Meeting: using a data set that will be provided by the lecturers the students will formulate and evaluate informative hypotheses in the context of ANOVA using Confirmatory ANOVA. Don't forget to use this and the other lab meetings to prepare for the grading session. This lab meeting will also be used to ensure that Confirmatory ANOVA and Winbugs/Openbugs work properly on your laptop.

### **Day 2: Introduction to the Key Concepts and Formulas of Bayesian Statistics – Introduction to OpenBUGS.**

Using simple models for binomial data a technical introduction of the key concepts in Bayesian statistics will be given. This day consists of three lectures and a lab meeting in which the students get acquainted with OpenBUGS and practice what they have learned during the lectures.

Lecture 1: Using an example in which the data have a binomial distribution, key concepts in Bayesian statistics will be introduced: prior distribution, density of the data, posterior distribution, expected a posteriori estimates, credibility intervals and sampling from the posterior distribution.

Lecture 2: The posterior predictive distribution will be discussed. This distribution lies at the heart of the Bayesian equivalent of classical hypothesis testing: posterior predictive p-values. The pros and cons of posterior predictive compared to classical p-values will be discussed.

Lecture 3: It will be elaborated how historical data can be used to specify informative prior distributions.

Lab Meeting: The students will learn to work with OpenBUGS. This will be done by means of an application of the key concepts and formulas of Bayesian statistics on data that have a binomial distribution.

### **Day 3: Bayesian Estimation: the Gibbs sampler and the Metropolis-Hastings Algorithm – Introduction to Winbugs**

Yesterday it was shown that it is easy to sample from posterior distributions that are based on simple binomial densities for the data. However, for more complicated models/densities direct sampling is often not possible. In three lectures and one lab meeting it will be explained how this problem can be solved.

Lecture 1: The Gibbs sampler: sampling from a multivariate posterior distribution if the conditional densities are known. Convergence issues when Markov chain Monte Carlo methods are used to sample from a posterior distribution.

Lecture 2: The augmented Gibbs sampler with applications to latent variable models and missing data problems.

Lecture 3: The Metropolis Hastings algorithm: sampling from a multivariate posterior distribution if the conditional densities are unknown.

Lab Meeting: OpenBUGS will be used to construct a Gibbs sampler for the posterior distribution of a logistic regression model. Furthermore, for a person with known predictors but unknown outcome, a prediction of and credibility interval for the unknown outcome will be obtained using OpenBUGS.

### **Day 4: Bayesian Model Selection: the Bayes Factor**

Today the Bayesian alternative for hypothesis testing will be discussed: Bayesian model selection. There will be three lectures and a lab meeting that is a continuation of Day 1 of this course.

Lecture 1: Key concepts in Bayesian model selection will be introduced: marginal likelihood, dependence of the marginal likelihood on the prior distribution, the choice of prior distributions if model selection is the goal, the Lindley paradox, the Bayes factor and posterior probabilities. This lecture will provide the technical foundation under the material presented during Day 1.

Lecture 2: Bayesian thinking will be elaborated. This will involve a comparison of classical and Bayesian probability, a comparison of hypothesis testing and model selection, and an example of Bayesian model selection.

Lecture 3: The Bayes factor for the evaluation of inequality constrained models will be elaborated. This is a continuation of the discussion on Day 1. Special attention will be given to the specification of prior distributions using training data and prior sensitivity.

Lab Meeting: Using OpenBUGS the participants will repeat the exercise that was executed on Day 1 using Confirmatory Anova.

## Day 5: Bayesian Model Selection: DIC

Today an alternative for the Bayes factor will be discussed: the DIC. There will be one lecture, a lab meeting that is a continuation of Day 3 of this course, and a session during which the performance of the participants will be graded.

Lecture 1: In this lecture the ideas underlying and the applications of the DIC will be discussed.

Lecture 2: The derivation of the DIC.

Lab Meeting: Using WinBUGS the students will practice using the DIC in the context of the selection of predictors for a logistic regression model.

Grading Session: See the section 'Grading'. The grading session will occupy most of the afternoon.

## References

### Contained in the Reader

Hoijtink, H. (2009). Bayesian Data Analysis. In: R.E. Millsap and A. Maydeu-Olivares, *The SAGE Handbook of Quantitative Methods in Psychology*. London: SAGE.

Klugkist, I., Laudy, O. and Hoijtink, H. (2005). Inequality Constrained Analysis of Variance: A Bayesian approach. *Psychological Methods*, 4, 477-493.

### Extra Reading for Day 1

Goodman, S.N. (1999). Toward evidence-based medical statistics. 1: The p-value fallacy. *Annals of Internal Medicine*, 130(12):995.

Goodman, S.N. (1999). Toward evidence-based medical statistics. 2: The Bayes Factor. *Annals of Internal Medicine*, 130(12):1005.

### *An application of Bayesian Statistics*

Van Well, S., Kolk, A.M. and Klugkist, I.G. (2008). Effects of Sex, Gender Role Identification, and Gender relevance of Two Types of Stressors on Cardiovascular and Subjective Responses: Sex and Gender Match/Mismatch Effects. *Behavior Modification*, 32, 427-449.

### Extra Reading for Day 2

Gelman, A., Meng, X. and Stern, H.S. (1996). Posterior predictive assessment of model fitness via realized discrepancies (with discussion). *Statistica Sinica*, 6, 733-760.

Greenland, S. (2006). Bayesian perspectives for epidemiological research: I. foundations and basic methods. *International Journal of Epidemiology*, 35(3):765-775.

### Extra Reading for Day 3

Zeger, S.L. and Karim, M.L. (1991). Generalized linear models with random effects: a Gibbs sampling approach. *Journal of the American Statistical Association*, 86, 79-86.

Greenland, S. (2007). Bayesian perspectives for epidemiological research. ii. regression analysis. *International Journal of Epidemiology*, 36(1):195-202.

### *An application of Bayesian Statistics*

Cerdá, M., Tracy, M, Messner, S.F., Vlahov, D., Tardiff, K., and Galea, S. (2009). Misdemeanor policing, physical disorder, and gun-related homicide: A spatial analytic test of "Broken-Windows" theory. *Epidemiology*, 20: 533-541

### Extra Reading for Day 4

Berger, J.O. and Perricchi, L. (2004). Training samples in objective Bayesian model selection. *The Annals of Statistics*, 32, 841-869.

Chib, S. (1995). Marginal likelihood from the Gibbs output. *Journal of the American Statistical Association*, 90, 1313-1321.

Kass, R.E. and Raftery, A. (1995). Bayes factors. *Journal of the American Statistical Association*, 90, 773-795.

**Extra Reading for Day 5**

Hamaker, E. L., Van Hattum, P., Kuiper, R. M. & Hoijtink, H. (in press). Model selection based on information criteria in multilevel modeling. In K. Roberts & J. Hox (Eds). *Handbook of Advanced Multilevel Analysis*. Taylor and Francis.

**Books**

Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B. (2004) *Bayesian Data Analysis*. London: Chapman and Hall.

Hoijtink, H., Klugkist, I. and Boelen, P. (2008). *Bayesian evaluation of informative hypotheses..* New York: Springer.

Lynch, S.M. (2007). *Introduction to applied Bayesian statistics and estimation for social scientists*. New York: Springer.

Ntzoufras, I. (2009). *Bayesian Modeling Using WinBUGS*. New York: Wiley.